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A FORM OF THE PARTICULAR SOLUTION OF THERMOELASTICITY EQUATIONS FOR TRANSVERSELY ISOTROPIC BODIES*

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A new representation is proposed for the particular solution of equations of linear uncoupled quasistatic thermoelasticity in displacements for transversely isotropic bodies. It contains two functions in a symmetric manner, which are determined independently of each other and satisfy equations that reduce to Poisson's equations by affine transformations of coordinates. In the isothermal case, the representation in question reduces to the well-known Elliott solution /1/. Cases of equal and unequal roots of the characteristic equation are considered separately. The representation obtained for the particular solution is more preferable from the viewpoint of satisfying the Sternberg criteria as compared with those known earlier. The Nowacki solution /2/ is expressed in a sufficiently complex manner in terms of one auxiliary function satisfying a fourth-order inhomogeneous partial differential equation. The representation from /3/, which is an extension of the frequently utilized Singh solution /4, 5/ to the non-axisymmetric case, contains two functions that satisfy the combined inhomogeneous second-order differential equations. By using the proposed solution a two-dimensional integral equation of the first kind is obtained for the contact pressure under a heated rigid stamp of arbitrary planform, implanted in a transversely-isotropic elastic half-space without friction. An exact analytic solution of the integral equation mentioned is constructed for a stamp of elliptical planform.

1. We select a rectangular coordinate system x_i in such a manner that the plane of isotropy of the transversely-isotropic material agrees with the x_1x_2 coordinate plane. The equations of linear uncoupled quasistatic thermoelasticity in terms of displacements for transversely-isotropic materials here take the form /2/

$$\begin{gathered}c_{11}u_{m,mm}+{}^1/_2(c_{11}-c_{12})u_{m,nn}+c_{44}u_{m,33}+{}^1/_2(c_{11}+c_{12})u_{n,mn}+\\(c_{13}+c_{44})u_{3,m3}=b_1T_{,m}\\c_{44}\Delta_2u_3+c_{33}u_{3,33}+(c_{13}+c_{44})(u_{1,13}+u_{2,23})=b_2T_{,3}\\b_1=(c_{11}+c_{12})\alpha_1+c_{13}\alpha_2,\quad b_2=2c_{13}\alpha_1+c_{33}\alpha_2\end{gathered}\tag{1.1}$$

where u_i are the components of the displacement vector, c_{ij} are stiffness coefficients in abbreviated notation (the formulas connecting the stiffness coefficients with the technical elastic constants are contained in /5/), T is the temperature measured from the initial value corresponding to vanishing stresses in the body and determined in the uncombined problem independently of the displacement field, α_1 is the coefficient of linear expansion in the x_1 and x_2 directions, α_2 in the x_3 direction, and Δ_2 is the two-dimensional Laplace operator in the variables x_1 and x_2.

The stress tensor components are determined in terms of the displacement vector and the temperature field by relationships

$$\begin{gathered}\sigma_{mm}=c_{11}u_{m,m}+c_{12}u_{n,n}+c_{13}u_{3,3}-b_1T\\\sigma_{33}=c_{13}(u_{1,1}+u_{2,2})+c_{33}u_{3,3}-b_2T\\\sigma_{m3}=c_{44}(u_{3,m}+u_{m,3})\\\sigma_{12}={}^1/_2(c_{11}-c_{12})(u_{2,1}+u_{1,2})\end{gathered}\tag{1.2}$$

The subscripts after the comma in (1.1) and (1.2) and everywhere later denote differentiation with respect to the corresponding coordinates, summation is not performed over repeated subscripts, and the subscripts m and n take the values 1 and 2, where $m\neq n$ in the limits of the separate expressions.

The stiffness coefficients satisfy the following inequalities /2/:

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$$c_{11} > 0, \quad c_{11} > c_{12}, \quad c_{44} > 0, \quad c_{33}(c_{11} + c_{12}) > 2c_{13}^2$$

that result from the condition of positive-definiteness of the specific strain elastic energy.

We will show that the particular solution of the inhomogeneous system (1.1) can be represented in the form

$$u_m^* = \varphi_{1,m} + \varphi_{2,m}, \quad u_3^* = h_1\varphi_{1,3} + h_2\varphi_{2,3} \tag{1.3}$$

where the functions φ_m (by analogy with the Goodier solution for an isotropic material /6/, it is natural to call them generalized displacement potentials) are solutions of the following similar equations:

$$\Delta_2\varphi_m + k_m\varphi_{m,33} = \beta_m T \tag{1.4}$$

and h_m, k_m and β_m are as yet undetermined constants.

Substituting (1.3) into (1.1) we find that the thermoelastic equilibrium equations in displacements are satisfied identically if the functions φ_m satisfy, in addition to (1.4), the system of equations

$$\sum_{j=1}^{2} \{c_{11}\Delta_2\varphi_j + [c_{44} + h_j(c_{13} + c_{44})]\varphi_{j,33}\} = b_1 T$$

$$\sum_{j=1}^{2} \{[c_{13} + (1 + h_j)c_{44}]\Delta_2\varphi_j + h_j c_{33}\varphi_{j,33}\} = b_2 T$$

which can, by using (1.4), be reduced to the form

$$\sum_{j=1}^{2} t_{mj}^*\varphi_{j,33} = t_{m3}^* T \quad (m = 1, 2) \tag{1.5}$$

$$t_{1m}^* = c_{44} + h_m(c_{13} + c_{44}) - k_m c_{11}$$

$$t_{13}^* = b_1 - (\beta_1 + \beta_2)c_{11}$$

$$t_{2m}^* = h_m c_{33} - k_m[c_{13} + (1 + h_m)c_{44}]$$

$$t_{23}^* = b_2 - \sum_{j=1}^{2}\beta_j[c_{13} + (1 + h_j)c_{44}]$$

The system (1.5) is satisfied identically if the constants h_m, k_m, and β_m are selected in such a manner that the relationships $t_{ni}^* = 0$ $(n = 1, 2;\ i = 1, 2, 3)$, which are a system (generally non-linear) of algebraic equations in the constants mentioned, are satisfied. Solving this system, we find that the constants k_m and β_m are expressed in terms of h_m according to the formulas

$$c_{11}k_m = c_{44} + (c_{13} + c_{44})h_m \tag{1.6}$$

$$c_{11}c_{44}(h_n - h_m)\beta_m = b_1[c_{13} + (1 + h_n)c_{44}] - b_2 c_{11}$$

while the constants h_m are the roots of the following characteristic equation

$$h^2 + \left[2 + \frac{c_{13}^2 - c_{11}c_{33}}{c_{44}(c_{13} + c_{44})}\right]h + 1 = 0 \tag{1.7}$$

and are determined by the formulas

$$h_m = 1 + C[A + (-1)^m D^{1/2}] \tag{1.8}$$

$$A = c_{11}c_{33} - (c_{13} + 2c_{44})^2, \quad D = AB$$

$$B = c_{11}c_{33} - c_{13}^2, \quad C = 2(B - A)^{-1}$$

Therefore, the particular solution of the inhomogeneous system of Eqs.(1.1) allows of representation in the form (1.3), (1.4) and the constants h_m, k_m, and β_m are determined by the relationships (1.6) and (1.8). In a special case (for $T \equiv 0$) the representation mentioned agrees with the well-known Elliott solution /1/.

Substituting (1.3) into (1.2) and using (1.4), we obtain the following representation of the stress tensor components in terms of the generalized displacement potentials:

$$\sigma_{mm}^* = -\sum_{j=1}^{2}[(c_{11} - c_{12})\varphi_{j,nn} + c_{44}(1 + h_j)\varphi_{j,33}] \tag{1.9}$$

$$\sigma_{33}^* = -c_{44}\Delta_2\sum_{j=1}^{2}(1 + h_j)\varphi_j$$

$$\sigma_{m3}^* = c_{44}\sum_{j=1}^{2}(1 + h_j)\varphi_{j,m3}, \quad \sigma_{12}^* = (c_{11} - c_{12})\sum_{j=1}^{2}\varphi_{j,12}$$

2. From the inequalities that the stiffness coefficients satisfy, it follows that the constant B is always positive. Consequently, the type of roots of the characteristic Eq. (1.7) is determined by the sign of the constant A: for $A > 0$ we have two different real roots of (1.7), for $A < 0$ two complex-conjugate roots, and for $A = 0$ two equal roots $h_m = 1$. The case of equal roots requires separate examination since the constants β_m cannot here be determined directly from (1.6).

In the case of equal roots of the characteristic equation when

$$(c_{13} + 2c_{44})\, c_{11}^{-1} = c_{33}\, (c_{13} + 2c_{44})^{-1} \equiv k$$

it is possible, by following /7/, to change to four new independent elastic constants μ, λ, δ and γ: $c_{11} = (\lambda + 2\mu)\,\delta$, $c_{33} = (\lambda + 2\mu)\,\delta^{-1}$, $c_{44} = \mu$, $c_{13} = \lambda$, $c_{11} - c_{12} = 2\gamma\mu$. Here $A \equiv 0$ and $k = \delta^{-1}$. Setting $\delta = \gamma = 1$ and $\alpha_1 = \alpha_2 = \alpha$, we emerge at the case of an isotropic material with Lamé constants μ and λ and coefficient of linear expansion α.

We seek the particular solution of (1.1) in the case of equal roots of the characteristic equation in the form

$$u_m^* = \psi_{1,m} + x_3\psi_{2,m}, \quad u_3^* = \psi_{1,3} + x_3\psi_{2,3} - p\psi_2 \tag{2.1}$$

where the functions ψ_m satisfy the equations

$$\begin{aligned} \Delta_2\psi_1 + k\psi_{1,33} &= \beta T - q x_3 T_{,3} \\ \Delta_2\psi_2 + k\psi_{2,33} &= q T_{,3} \end{aligned} \tag{2.2}$$

Substituting (2.1) into (1.1) and using (2.2) we find (we omit the calculations analogous to those made above for the unequal root case) that the constants p, q, and β are determined by the following relationships:

$$p = (c_{13} + 3c_{44})\,(c_{13} + c_{44})^{-1}, \quad \beta = b_1 c_{11}^{-1}$$
$$q = (b_2 - kb_1)\,[(1 - p)\, kc_{11}]^{-1}$$

Changing to an isotropic material in (2.1) and (2.2), we obtain

$$k = 1, \ q = 0, \ \beta = \alpha\,(3\lambda + 2\mu)\,(\lambda + 2\mu)^{-1}$$

so that, without loss of generality, $\psi_2 \equiv 0$, and the mentioned representation reduces to the Goodier solution /6/, while the function ψ_1 reduces to the classical displacement potential.

In the case of equal roots of the characteristic equation the stress tensor components are expressed in terms of the generalized displacement potentials ψ_m as follows:

$$\begin{aligned} \sigma_{mm}^* &= -(c_{11} - c_{12})\,(\psi_{1,nn} + x_3\psi_{2,nn}) - \\ &\quad 2c_{44}\,(\psi_{1,33} + x_3\psi_{2,33}) + c_{13}\,(1 - p)\,\psi_{2,3} \\ \sigma_{33}^* &= (c_{13} - b_2\beta^{-1})\,(\Delta_2\psi_1 + x_3\Delta_2\psi_2) + (c_{33} - kb_2\beta^{-1})\,(\psi_{1,33} + \\ &\quad x_3\psi_{2,33}) + c_{33}\,(1 - p)\,\psi_{2,3} \\ \sigma_{m3}^* &= c_{44}\,[2\psi_{1,m3} + 2x_3\psi_{2,m3} + (1 - p)\,\psi_{2,m}] \\ \sigma_{12}^* &= (c_{11} - c_{12})\,(\psi_{1,12} + x_3\psi_{2,12}) \end{aligned} \tag{2.3}$$

3. By using the obvious affine coordinate transformations the equations in the generalized displacement potentials $\varphi_m\ (A \neq 0)$ or $\psi_m\ (A = 0)$ are reduced to Poisson's equations. The particular solutions of these equations can be selected in the form of integrals of potential type, for instance, the functions φ_m allow of such a representation:

$$\varphi_m(x_i) = -\frac{\beta_m}{4\pi} \iiint \frac{T(x_i')\, dx_1'\, dx_2'\, dx_3'}{\{k_m\,[(x_1 - x_1')^2 + (x_2 - x_2')^2] + (x_3 - x_3')^2\}^{1/2}}$$

where the integration is over the domain occupied by the elastic material.

By comparison with the forms proposed earlier /2, 3/ for the solution of (1.1), the representation (1.3), (1.4) is most preferable from the viewpoint of satisfying criteria proposed by Sternberg (these criteria are listed in /8/) to estimate methods of introducing the stress functions.

A more general solution of the thermoelastic equilibrium Eqs. (1.1) can be obtained by setting

$$u_1 = u_1^* + \omega_{,2}, \quad u_2 = u_2^* - \omega_{,1}, \quad u_3 = u_3^* \tag{3.1}$$

where u_i^* are determined by (1.3) in the case of unequal roots of the characteristic equation and by (2.1) in the case of equal roots, while the function ω introduced in /9, 10/, satisfies the homogeneous equation $\Delta_2\omega + 2c_{44}\,(c_{11} - c_{12})^{-1}\omega_{,33} = 0$ and is independent of the temperature field.

The representation proposed by Nowacki /2/ for the particular solution of (1.1) in terms of one function satisfying a fourth-order inhomogeneous partial differential equation can be obtained from the solution (1.3), (1.4). To this end, we introduce a new function Ψ by using the relationships

$$\varphi_m = \beta_m (\Delta_2\Psi + k_n\Psi_{,33}) \tag{3.2}$$

Thereby Eqs.(1.4) for the generalized displacement potentials φ_m are reduced to one fourth-order equation in the function Ψ, namely

$$\left(\Delta_2 + k_1 \frac{\partial^2}{\partial x_3^2}\right)\left(\Delta_2 + k_2 \frac{\partial^2}{\partial x_3^2}\right)\Psi = T$$

Representations of the displacement vector and stress tensor components in terms of the function Ψ can be obtained by substituting (3.2) into (1.3) and (1.9).

4. The representation (3.1) enables us to construct solutions of the boundary value problems of the theory of thermal stresses for transversely-isotropic bodies in the form of the superposition of solutions of independent equations that are reduced to Poisson's equations by using affine coordinate transformations.

As an illustration we consider the non-axisymmetric mixed boundary value problem of the pressure of an absolutely stiff heated stamp of arbitrary planform on a transversely-isotropic elastic half-space $x_3 \geqslant 0$ when there are no friction forces.

The mechanical boundary conditions of the problem in question have the form

$$\begin{aligned} &\sigma_{m3}(\mathbf{x}, 0) = 0, \quad 0 \leqslant x < \infty \\ &u_3(\mathbf{x}, 0) = f(\mathbf{x}), \quad \mathbf{x} \in S \\ &\sigma_{33}(\mathbf{x}, 0) = 0, \quad \mathbf{x} \notin S \\ &\mathbf{x} = (x_1, x_2), \ x = |\mathbf{x}| \equiv (x_1^2 + x_2^2)^{1/2} \end{aligned} \tag{4.1}$$

where $f(\mathbf{x})$ is a given function (specified apart from the displacement parameters of the stamp as a solid body), describing the shape of the stamp base, and S is the contact area.

When there are no heat sources the stationary temperature field satisfies the equation /5/

$$\Delta_2 T + \varkappa^2 T_{,33} = 0 \tag{4.2}$$

where $\varkappa^2$ is the ratio between the thermal conductivity in the x_3 direction and the thermal conductivity in the x_1 and x_2 directions.

To be specific, we consider temperature boundary conditions of the type *a* in the classification from the paper /11/, in which four kinds of idealized conditions are mentioned for contact problems, namely ($t(\mathbf{x})$ is a given function)

$$T(\mathbf{x}, 0) = t(\mathbf{x}), \quad \mathbf{x} \in S; \quad T(\mathbf{x}, 0) = 0, \quad \mathbf{x} \notin S \tag{4.3}$$

Relationships (4.1) and (4.3) should be supplemented by standard equilibrium conditions for the stamp and attenuation of the thermoelastic field at infinity /12, 13/.

We will examine the case of unequal roots of the characteristic equation. For vanishing shear stresses on the half-space boundary, it is possible to set $\omega \equiv 0$, in (3.1) without loss of generality; consequently, we use the representation (1.3) directly while omitting the asterisk in the notation for the displacement and stress.

We introduce the two-dimensional Fourier integral transform operator

$$\begin{aligned} &F\{\varphi(\mathbf{x}, x_3)\}(\xi, x_3) \equiv \varphi^F(\xi, x_3) = \iint_{-\infty}^{\infty} \varphi(\mathbf{x}, x_3) \exp(i\xi\cdot\mathbf{x})\, d\mathbf{x} \\ &\xi = (\xi_1, \xi_2), \quad \xi\cdot\mathbf{x} = \xi_1 x_1 + \xi_2 x_2 \end{aligned}$$

acting on (4.2) and (1.4), and by solving the ordinary differential equations hence obtained and taking account of the conditions at infinity, we find ($E_m(\xi)$ are arbitrary functions)

$$\begin{aligned} &T^F(\xi, x_3) = T^F(\xi, 0)\exp(-\xi z), \quad \xi = |\xi| \\ &\varphi_m^F(\xi, x_3) = E_m(\xi)\exp(-\xi z_m) + \\ &\quad \beta_m\varkappa^2(k_m - \varkappa^2)^{-1}\xi^{-2}T^F(\xi, 0)\exp(-\xi z) \\ &z = x_3/\varkappa, \quad z_m = x_3/k_m^{1/2} \end{aligned} \tag{4.4}$$

Applying a Fourier transform to (1.3) and (1.9), substituting (4.4) into the relationships obtained and setting $x_3 = 0$, we arrive, in particular, at the following representations:

$$u_3^F(\xi, 0) = -\sum_{j=1}^{2} h_j[\xi E_j(\xi) k_j^{-1/2} + \varkappa\xi^{-1}T^F(\xi, 0) r_j] \tag{4.5}$$

$$\sigma_{m3}^F(\xi, 0) = i\xi_m c_{44} \sum_{j=1}^{2} (1 + h_j)[\xi E_j(\xi) k_j^{-1/2} + \varkappa\xi^{-1} T^F(\xi, 0) r_j]$$
$$\sigma_{33}^F(\xi, 0) = \xi^2 c_{44} \sum_{j=1}^{2} (1 + h_j)[E_j(\xi) + \varkappa^2\xi^{-2} T^F(\xi, 0) r_j]$$
$$r_m = \beta_m (k_m - \varkappa^2)^{-1}$$

Using the first boundary condition (4.1), we find a relation between the functions $E_m(\xi)$ from the second relationship of (4.5)

$$E_2(\xi) = -\frac{(1+h_1) k_2^{1/2}}{(1+h_2) k_1^{1/2}} E_1(\xi) - \frac{\varkappa k_2^{1/2}}{(1+h_2)\xi^2} T^F(\xi, 0) \sum_{j=1}^{2} (1 + h_j) r_j \tag{4.6}$$

Substituting (4.6) into the first and third equations of (4.5) and eliminating the function $E_1(\xi)$ from the relationships obtained, we establish a formula connecting the Fourier transforms of the normal displacements and the normal stresses at points of the plane $x_3 = 0$

$$u_3^F(\xi, 0) = H^*\xi^{-1}\sigma_{33}^F(\xi, 0) + G^*\xi^{-1}T^F(\xi, 0) \tag{4.7}$$
$$H^* = (k_1^{1/2} + k_2^{1/2}) c_{11} (c_{13}^2 - c_{11}c_{33})^{-1}$$
$$G^* = \varkappa (k_1^{1/2} - k_2^{1/2})^{-1} [\beta_1 (1 - h_1)(k_1^{1/2} + \varkappa)^{-1} - \beta_2 (1 - h_2)(k_2^{1/2} + \varkappa)^{-1}]$$

Acting on (4.7) with the inverse Fourier transform operator, and taking account of the third boundary condition (4.1) and the boundary conditions (4.3), we find

$$u_3(\mathbf{x}, 0) = H \iint_S \frac{\sigma(\mathbf{x}')\, d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|} + G \iint_S \frac{t(\mathbf{x}')\, d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|} \tag{4.8}$$
$$H = -(2\pi)^{-1} H^*, \quad G = (2\pi)^{-1} G^*$$

where $\sigma(\mathbf{x}) = -\sigma_{33}(\mathbf{x}, 0)$ is the contact pressure.

Satisfying the second boundary condition (4.1) by using (4.8), we arrive at a two-dimensional integral equation of the first kind in the contact pressure

$$H \iint_S \sigma(\mathbf{x}') |\mathbf{x} - \mathbf{x}'|^{-1} d\mathbf{x}' = f(\mathbf{x}) - G\theta(\mathbf{x}), \ \mathbf{x} \in S \tag{4.9}$$
$$\theta(\mathbf{x}) = \iint_S t(\mathbf{x}') |\mathbf{x} - \mathbf{x}'|^{-1} d\mathbf{x}'$$

which reduces to a well-known equation in the isothermal case (when $\theta(\mathbf{x}) \equiv 0$) /13/.

5. The contact problem of transversely-isotropic thermoelasticity for a circular stamp has been investigated fairly completely /5/, certain results on the thermal contact problem of isotropic elasticity theory are presented /11, 14/ for an elliptical stamp, and the corresponding isothermal case is studied in /12, 13/.

We consider below the contact problem for a heated stamp of elliptical planform with a polynomial base shape pressed into a transversely-isotropic elastic half-space (the principal vector and principal moments of the forces applied to the stamp are considered given). In this case

$$S = \left\{(\mathbf{x}, x_3): \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} \leqslant 1, x_3 = 0\right\}, \quad f(\mathbf{x}) = \sum_{i+j=0}^{l_1} f_{ij} x_1^i x_2^j$$

where a_1 and a_2 are semi-axes of the ellipse, $f(\mathbf{x})$ is a polynomial of arbitrary degree l_1 in x_1 and x_2, where the coefficients f_{ij} are given for $i + j > 1$, while f_{00}, f_{10}, and f_{01} are the translational displacement and projections of the stamp rotation vector, not known in advance.

It can be shown that if the given temperature distribution over the contact area S has the form

$$t(\mathbf{x}) = v^{-1/2}(\mathbf{x}) \sum_{i+j=0}^{l_2 - 2k} t_{ij} x_1^i x_2^j, \quad v(\mathbf{x}) = 1 - \frac{x_1^2}{a_1^2} - \frac{x_2^2}{a_2^2} \tag{5.1}$$

where l_2 is an arbitrary integer, k is any of the numbers $0, 1, 2, \ldots, [l_2/2]$, and $[r]$ is the integer part of the number r, then

$$\theta(\mathbf{x}) = \sum_{i+j=0}^{l_2} \theta_{ij} x_1{}^i x_2{}^j$$

where the coefficient θ_{ij} are expressed in a known manner in terms of the constants t_{ij} /15/.

Under the assumptions made above, the integral Eq.(4.9) contains a polynomial of degree $l = \max\{l_1, l_2\}$ in x_1 and x_2 on the right side

$$H \iint_S \sigma(\mathbf{x}') |\mathbf{x} - \mathbf{x}'|^{-1} d\mathbf{x}' = \sum_{i+j=0}^{l} q_{ij} x_1{}^i x_2{}^j, \ \mathbf{x} \in S \tag{5.2}$$

whose coefficients q_{ij} are determined in an obvious manner in terms of the constants f_{ij} and θ_{ij}.

Setting

$$\sigma(\mathbf{x}) = v^{-1/2}(\mathbf{x}) \sum_{i+j=0}^{l} \tau_{ij} x_1{}^i x_2{}^j \tag{5.3}$$

we reduce (5.2) to a system of $^1/_2(l+1)(l+2)$ linear algebraic equations in $3 + {}^1/_2(l+1)(l+2)$ unknown constants f_{00}, f_{10}, f_{01} and τ_{ij}. The stamp equilibrium conditions yield three additional algebraic equations for τ_{ij}. The explicit form of all these algebraic equations is mentioned in /15/.

Therefore, if a stamp of elliptical planform with a sharp edge has a polynomial base and the temperature distribution over the contact area is described by (5.1), then the contact pressure under the stamp has the form (5.3) and the coefficients τ_{ij} are determined from the corresponding system of linear algebraic equations.

The result established extends the well-known Galin theorem on the functional form of the contact pressure under a stamp of elliptical planform /12/ to the case of transversely-isotropic thermoelasticity.

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